Design of Fractional order controller using Particle Swarm Optimization for a four tank Level process

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ABSTRACT

Fractional calculus (FC) is widely used in most areas of science and engineering, being recognized its ability to yield a superior modelling and control in many dynamical systems. This paper presents the applications of a Fractional Order PID (FOPID) controller in the area of Process Control for a multivariable system of coupled tanks to evaluate the use of intelligent technique with FOPID control techniques. To design FOPID controller is to determine the two important parameters $\lambda$ (integrator order) and $\mu$ (derivative order) using Particle Swarm Optimization (PSO). In this article that the response and performance of FOPID controller is compared with closed loop conventional PID controller and PSO based PID Controller. In all the cases the FOPID much better than integer order PID controller for the same system.

Keywords - Level control, multivariable systems, FOPID, $\lambda$ and $\mu$ analysis, Particle Swarm Optimization

1. INTRODUCTION

For linear systems, the Proportional Integral Derivative (PID) controller has been widely used in industrial control processes because it has a simple structure and robust performance, and it is easily tuned in a wide range of operating conditions [1]. In spite of the fact that control theory has been developed significantly, PID controllers are still used for many industrial applications such as process controls, motor drivers, flight control, and instrumentation.

Fractional order dynamic systems and controllers have been increasing in interest in many areas of science and engineering in the last few years. Fractional order controllers are described by fractional order deferential equations. Expanding derivatives and integrals to fractional orders can adjust the control system’s frequency response directly and continuously. Controllers consisting of fractional order derivatives and integrals have been used in industrial applications [2] and various fields such as power electronics [3], system identification [4], robotic manipulators [5], irrigation canal control [6], mechatronics systems [7,8], and heat diffusion systems [9]. It should be noted that there are a growing number of physical systems whose behaviour can be compactly determined using the fractional order system theory and can be controlled with fractional order proportional-integral-derivative (FOPID) controllers [10], even if the system has unstable or time delay behaviours [11].

There are many aspects that should be taken into account when designing these controllers. In the FOPID controller, the 5 parameters ($K_F$, $K_i$, $K_d$, $\lambda$, $\mu$) need to be tuned based on some design specifications. The desired specifications for the controllers are usually to achieve robust to load disturbances, high frequency noise, and uncertainties of the plant model. Taking into account all of the constraints in the tuning method of the FOPID controller, the optimal set of values for $K_F$, $K_i$, $K_d$, $\lambda$ and $\mu$ can be found.

In this paper it is considered a 4 level tank process, for the analysis of Fractional order PID controllers. Where the first three tanks have constant cross section of 60 cm$^2$ and the fourth tank has a variable section, varying between 60cm$^2$ and 121.2cm$^2$. The maximum allowed level is 49.5cm. Tanks 1 and 3 have water input and output driven by electric pumps, tanks 2 and 4 have 2mm water outlets into a reservoir. Coupling valves between adjacent tanks (a 2mm groove with configurable height) determine the flow parameters of the process. Each tank has is equipped with a pressure level sensor. The contribution of this work between adjacent tanks (a 2mm groove with configurable height) determine the flow parameters of the process.

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It is quite difficult to optimize the parameters of the FOPID controller in linear and nonlinear systems. There is a need for an effective and efficient global approach to optimize these parameters automatically. We used particle swarm optimization (PSO) [12-15] have been proposed to optimize the parameters of the several controllers. PSO is a relatively new evolutionary algorithm that may be used to find optimal or near optimal solutions in a large search space. The PSO algorithm is especially useful for parameter optimization in continuous, multidimensional search spaces. The PSO method can generate a high quality solution within a shorter calculation time and it tends to converge very fast compared to other stochastic methods. Moreover, it is implemented easily in most of the programming languages.
2. CONVENTIONAL PID CONTROLLER

PID is the most widely-used type of controller for industrial applications. And exhibit robust performance over a wide range of operating conditions. The three main parameters involved are Proportional (P), Integral (I) and Derivative (D). The proportional part is responsible for following the desired set-point, while the integral and derivative part account for the accumulation of past errors and the rate of change of error in the process respectively.

![PID Block-diagram]

Fig.1 Block-diagram of PID

PID controller’s algorithms are mostly used in feedback loops. PID controllers can be implemented in many forms. It can be implemented as a stand-alone controller or as part of Direct Digital Control (DDC) package or even Distributed Control System (DCS). A PID controller has three tuning parameters. If these are adjusted in an ad hoc fashion, it may take a while for satisfactory performance to be obtained.

Table.1 PID tuning method

Also, each tuning technician will end up with a different set of tuning parameters. There is plenty of motivation then to develop an algorithmic approach to controller tuning. The Ziegler-Nichols closed-loop tuning technique was perhaps the first rigorous method to tune PID controllers. The technique is not widely used today because the closed-loop behaviour tends to be oscillatory and sensitive to uncertainty.

3. FRACTIONAL ORDER PID Controller

One of the possibilities for improvements in the quality and robustness of PID controllers is to use fractional order controllers with noninteger derivation and integration parts. The PI\(^D\) controller involving an integrator of order \(\lambda\) and a differentiator of order \(\mu\).

The differential equation of the PI\(^D\) controller is given as follows:

\[
u(t) = K_p e(t) + K_\lambda D^{-\lambda} e(t) + K_\mu D^{\mu} e(t)\tag{1}\]

The continuous transfer function of the FOPID controller is obtained by means of the Laplace transformation, as given by:

\[
G(s) = \frac{U(s)}{E(s)} = K_p + K_\lambda s^{-\lambda} + K_\mu s^{\mu}, \quad (\lambda, \mu > 0)\tag{2}
\]

For designing a FOPID controller, 3 parameters (\(K_p, K_i, K_d\)) and 2 orders (\(\lambda, \mu\)) with non-integers should optimally determine for a given system.

4. NON LINEAR LEVEL PROCESS OF 4 LEVEL TANK SYSTEM

The process investigated in this work is a multivariable system of interconnected tanks build for the evaluation of control techniques, figure 1. Three of these tanks possess dimensions of 49.5x10x6 cm and the fourth tank has a non uniform section. The transversal section of tanks 1, 2 and 3 is 60 cm\(^2\). The width of tank 4 until 14.7 cm height, representing 29% of the liquid level, has the same value, 60 cm\(^2\). Then it gets linearly wider up to 20.2 cm (31.65° aperture angle) with a transversal area changing from 60 cm\(^2\) to 121.2 cm\(^2\).

Tank 1, has a water input, pumped from reservoir by means of a pump u1, and also has a water output, q_{o1}. A second pump u2 drives water from 1 to the reservoir. Tank 2, has a water output, q_{o2}, passing through a puncture...
of 2mm, situated in the bottom of this tank to the reservoir. Tank 3, has water input, which is pumped from the reservoir by means of pump u3. Tank 3 also has a water exit, q_o3. Tank 3 has also the possibility to pump water out using pump u4. The contents of tank 4 reaches the reservoir through q_o4, a puncture of 2mm, situated in the bottom of this tank.

Fig.2 Schematic representation of the 4 level order liquid level process. Between the tanks a groove with approximately 2mm width and configurable height is used, that determines the interconnecting flow parameter. The pumps are driven by means of a power system commanded by a voltage between 0 and 10 VCC. Each tank is endowed with a pressure level sensor, able to measurement with good precision inclusive the maximum water column of 49.5 cm.

5. PARTICLE SWARM OPTIMIZATION

PSO is an evolutionary computational technique based on the movement and intelligence of swarms looking for the most fertile feeding location. A “swarm” is an apparently disorganized collection (population) of moving individuals that tend to cluster together, while each individual seems to be moving in a random direction. PSO uses a number of agents (particles) that constitute a swarm moving around in the search space looking for the best solution [16-18].

Each particle is treated as a point in an n-dimensional space and adjusts its “flying” according to its own flying experience, as well as the flying experience of other particles. Each particle keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) that has been achieved so far. This value is called pbest. Another best value called gbest is that obtained so far by any particle in the neighbours of the particle.

The PSO concept consists of changing the velocity (or acceleration) of each particle toward its pbest and the gbest position at each time step. Each particle tries to modify its current position and velocity according to the distance between its current position and pbest, and the distance between its current position and the gbest. At each step n, by using the individual best position, pbest, and global best position, gbest, a new velocity for the ith particle is updated by:

\[ V_i(n) = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \cdot \varphi \cdot (p_{best}(n) - P_{i}(n-1) + p_{best}(n) - P_{i}(n-1)) \]  

X is defined below:

\[ x = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \cdot \varphi \cdot \varphi > 4 \]

The velocity is confined within the range of \([-v_{max}, v_{max}]\). If the velocity violates these limits, it is forced to its proper values. Changing velocity in this way enables the ith particle to search around its local best position, pbest, and global best position, gbest. Based on the updated velocity, each particle changes its position as follows:

\[ P_{i}(n) = P_{i}(n-1) + V_{i}(n) \]

Fig.3 Tuning process of the FOPID controller parameters with PSO
6. MODELLING AND PARAMETER ESTIMATION OF 4 RD ORDER TANK SYSTEM

A nonlinear mathematical model for a similar four tank system used in the present study includes the disturbance effect of flows in and out of the system.

\[ A_1 \frac{d^4 q_1}{dt^4} = q_{11} - k_{51} \sqrt{h_1} - \text{signal}(h_1 - h_2)k_{12}\sqrt{h_1 - h_2} \]  

\[ A_2 \frac{d^4 q_2}{dt^4} = \text{signal}(h_2 - h_3)k_{12}\sqrt{h_1 - h_2} - k_{62} \sqrt{h_2} - \text{signal}(h_2 - h_3)k_{23}\sqrt{h_2 - h_3} \]  

\[ A_3 \frac{d^4 q_3}{dt^4} = \text{signal}(h_3 - h_4)k_{23}\sqrt{h_2 - h_3} - k_{63} \sqrt{h_3} - \text{signal}(h_3 - h_4)k_{34}\sqrt{h_3 - h_4} \]  

\[ A_4 \frac{d^4 q_4}{dt^4} = \text{signal}(h_4 - h_4)k_{34}\sqrt{h_3 - h_4} - k_{64} \sqrt{h_4} \]  

The valve \( k_{12} \) allow water to flow between tank 1 and tank 2. In the same way \( k_{23} \) and \( k_{34} \) shape the flow between tanks 2 and 3, and 3 and 4, respectively. The problem focused in this work is the height control of level 4 \( (h_4) \).

The outflows and the heights are thus defined as follows:

- \( q_{11} \) and \( q_{13} \) = inputs flows into tanks 1 and 3, [cm^3/s];
- \( q_{01} \) and \( q_{03} \) = output flows from tanks 1 and 3, [cm^3/s];
- \( q_{12}, q_{23} \) and \( q_{34} \) = interconnected flows between tanks 1-2, 2-3 and 3-4, [cm^3/s];
- \( q_{02} \) and \( q_{04} \) = output flows from tanks 2 and 4, through 2mm punctures [cm^3/s];
- \( h_1, h_2, h_3 \) and \( h_4 \) = water column height in tanks 1, 2, 3 and 4, [cm].

It is the area of the transversal section of tanks

\( k \) is the valve parameter between tanks \( k_{12}, k_{23} \) and \( k_{34} \) [cm^3/s]

7. SIMULATION RESULTS

The plant was identified by the training data generated by using Particle Swarm Optimization. After this, we analysis \( \lambda \) and \( \mu \) values and applied to the Fractional Order PID Controller of the 4rd order tank system to verify the response of the controller the configuration in figure 6 was used. All the simulations have been carried out in the SIMULINK of MATLAB software. As described, answers for some configurations of the interconnected water tanks had been obtained.

The following parameters were used:

\( q_{11} = 0 \) a 66 [cm^3/s]; \( q_{13} = 0 \) [cm^3/s]; \( h_4 = 0 \) a 46 [cm]; \( k_{12} = 0, k_{23} = 5, k_{34} = 5 \) [cm2,5/s]; \( k_{12} = 12, k_{23} = 2.5, k_{34} = 6 \) [cm2,5/s]

![Diagram of 4 Level Tank Blocks](image)

![Level Tank process with Conventional PID Controller](image)
Fig. 6 Level Tank Process with Fractional Order PID

Fig. 7 Conventional PID Controller for 4 level Tank Process

Fig. 8 FOPID Controller for 4 level Tank Process

Fig. 9 Conventional PID and Fractional Order PID controller Combined Performance

Fig. 10 Conventional PID Controller for 4 level Tank Process with 10% disturbance at 30 secs
CONCLUSION
This paper introduced an intelligent optimization method for FOPID controllers tuned with Particle Swarm Optimization. In order to evaluate the performance of the controller, the 4 tank liquid level process was done with MATLAB/Simulink. The robust design of the FOPID controller is difficult to compare to the PID controller, since the FOPID controller includes more parameters. All of the parameters related to the FOPID controller were determined using PSO. The performance of PSO and the conventional PID was compared with several simulation experiments. Considering all of the results from the simulation experiments, the FOPID-PSO controller can achieve good performance and robustness, superior to those obtained with the conventional PID controller. Moreover, PSO can achieve faster search speed and better solutions compared to the others. In addition, the FOPID-PSO controller enhanced the flexibility and stability of the PID controller.

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AUTHOR BIOGRAPHY

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