**Model-Free Sliding Mode Fuzzy Controller with Minimum Rule Base Implemented on Bus Suspension System**

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**ABSTRACT**

Although both fuzzy logic controller and sliding mode controller have admirable characteristics, they have some disadvantages when used individually. To eliminate these disadvantages, a sliding mode fuzzy controller (SMFC) is proposed and is applied to the bus suspension system. MATLAB/SIMULINK is used as the platform to do the experiments. The experimental results of the bus suspension system using sliding mode fuzzy controller (SMFC) are then compared with the experimental results of a fuzzy logic controller (FLC) and a PID controller. The simulation results illustrate that the SMFC has better performance compared with FLC and PID controller.

**Keywords:** Bus Suspension System, sliding mode control, fuzzy logic control, sliding mode fuzzy control, MATLAB/Simulink, chattering phenomenon, nonlinear control

1. INTRODUCTION

Due to increase in demand for better ride comfort and road holding performance, the interest in the field of controlling the suspension systems has been increasing in the recent decades. Numerous applications of different control strategies have been proposed for control of suspension systems. Control strategies such as adaptive control, linear quadratic Gaussian control, nonlinear control, and intelligent control [1]. In this paper a sliding mode fuzzy controller is introduced and is used to control the bus suspension system. Controllers have become an essential part of modern society. They are used in different applications such as the rocket fire, the space shuttle lifts off to earth orbit, a self-guided vehicle, and robots. The main purpose of designing a controller is to obtain a desired output with desired performance, with given specific inputs. Some measurements of performance are transient response, steady-state error, stability, robustness, and disturbance rejection [2]. Controllers are devices which can sense the information from the plant or process (for example a bus suspension system) and use this information to reach the desired performance [3]. Variable Structure Systems (VSS) with Sliding Mode Control (SMC) was first used by Russian researchers in sixties. Up until seventies, this new idea was only used by Russian researchers. In seventies, a book by Itkis [4] and a survey paper by Utkin [5] introduced this method worldwide. From then, Sliding Mode Controllers have been developed and were successfully implemented in different applications such as nonlinear control, MIMO, discrete-time models, large scale and Infinite-Dimensional systems [6]. Some applications of SMC are presented in [7] and [8]. By using discontinuous feedback control laws, SMC compels the system state to slide on a switching surface which is within the bounds of the state space and then controller tries to keep the system in the specified surface. One advantage of using Sliding Mode Controller is that by moving on the sliding surface, system becomes insensitive to some model uncertainties and perturbations, and the controller has acceptable performance even with these uncertainties [6]. Fuzzy logic was first introduced by Lotfi Zadeh in 1965[9] and since then it was used in numerous industrial applications such as in washing machines, elevators, trains, cranes, automotive industry, traffic control, and medical diagnosis [10]. Various combinations of fuzzy logic controller (FLC) with sliding mode controller (SMC) have been proposed in [11-12]. Sliding mode fuzzy controller (FSMC) uses SMC advantages like robustness, stability, and insensitivity against some uncertainties, and uses Fuzzy logic controller to eliminate disadvantages of SMC like chattering problem [7-8]. Various combinations of fuzzy controller with sliding mode controller has been proposed in [13-21]. In Table 1 the advantages and disadvantages of SMC, FLC, and SMFC are proposed.

<table>
<thead>
<tr>
<th>Controller’s name</th>
<th>Advantages</th>
<th>Disadvantages</th>
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<tbody>
<tr>
<td>FLC</td>
<td>simple implementation, flexibility</td>
<td>needs expert knowledge to optimize</td>
</tr>
<tr>
<td>SMC</td>
<td>insensitive to external disturbances, stable</td>
<td>chattering phenomenon, dynamic dependency</td>
</tr>
<tr>
<td>SMFC</td>
<td>insensitive to external disturbances, stable, flexible, Model-free</td>
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The objective of this paper is to design a sliding mode fuzzy controller (SMFC) and implement it on bus suspension system. The experimental results of the SMFC are then compared with the experimental results of FLC and PID controller. Rest of the paper is organized as follows.
In section 2, the dynamic formulation of the bus suspension system is presented. The fuzzy logic controller is presented in section 3. Section 4 describes the sliding mode fuzzy controller. Simulation, results and discussions are presented in section 5, and finally in section 6, the conclusion is presented.

2. DYNAMIC FORMULATION OF BUS SUSPENSION SYSTEM

In the present paper, a simple bus suspension system has been used as the plant. The bus suspension system is illustrated in Fig 1.

![Bus suspension system (1/4)](image)

Figure 1: Bus suspension system (1/4)

The dynamic equations of the system are obtained by using the Newton’s law.

\[
M_1 \ddot{y}_1 = -b_2 (\dot{y}_1 - \dot{y}_2) - k_1 (y_1 - y_2) + F \\
M_2 \ddot{y}_2 = -b_2 (\dot{y}_1 - \dot{y}_2) + k_1 (y_1 - y_2) + b_2 (F_r - \ddot{y}_2) + k_2 (F_r - y_2) - F
\]

where \(M_1\) is the body mass, \(M_2\) is the suspension mass, \(k_1\) is the spring constant of suspension system, \(k_2\) is the spring constant of wheel and tire, \(b_1\) is the damping constant of suspension system, \(b_2\) is the damping constant of wheel and tire, \(F\) is the control force, and \(F_r\) is the road profile.

3. SLIDING MODE CONTROLLER DESIGN

A sliding mode fuzzy controller is designed to control the suspension system. In order to design the controller, the dynamic equation of the system is rewritten as below [22-23].

\[
\dot{y}_p = f(y_p, F_r, t) + \Delta f(y_p, F_r, t) + K_c U
\]

where \(f(y_p, F_r, t)\) represents the nominal dynamic of the system and, \(\Delta f(y_p, F_r, t)\) represents the nonlinearities and dynamics which are caused by changes of the dynamic variables, \(F_r\) is the road profile input, \(U\) is the control efforts , and \(K_c\) is a constant with \(K_c \neq 0\), and \(f(.), \Delta f(.) \in R^{n \times m}, \Delta f \in R^{n \times m}, \text{and } U \in R^{n \times m} \).

The upper bound is defined as:

\[
|\Delta f(y_p, F_r, t)| \leq K_d
\]

where \(K_d\) is a positive known constant. \(d f = |f - \bar{f}| \leq K_f\) where \(K_f\) is a positive constant and \(\bar{f}\) is the nominal value of \(f\). The purpose of designing sliding mode controller is to get the state \(y\) for a desired state \(y_d\).

The tracking error is defined as:

\[
e = y_{\text{ref}} - y - \theta
\]

where \(y_{\text{ref}}\) is the desired vertical position. \(\theta\) is defined as:

\[
y = y_1 - y_2
\]

the sliding surface is calculated as below.

\[
S = (\dot{\theta} + \dot{\lambda} \dot{\theta})
\]

where \(\lambda\) is the sliding surface gain and is strictly positive which forces the system state to slide on a switching surface.

A possible solution to the sliding mode control is:

\[
u = u_{eq} + u_{dis}
\]

where

\[
u_{eq} = -\frac{1}{k_z}[f(y, \dot{y}, t) + \lambda \dot{\theta} - \dot{y}_d]
\]

and

\[
u_{dis} = -k_2 \dot{y}_d \text{sgn}(S)
\]

where \(\text{sgn}(\cdot)\) represents the sign function, and \(k_2\) is the switching gain and is a positive constant which satisfies the following condition:
\[ k_e \geq \eta + K_d + K_f \] 

where \( \eta \) is a positive constant. The chattering phenomenon happens because of the discontinuous part of the controller \( u_{dis} \). To eliminate the chattering problem, a boundary layer is set around the sliding surface [22-24].

In equation (10) the sign function is replaced with saturation function so:

\[ u_{dis} = -\frac{k_e}{\varepsilon} \text{sat}(\frac{e}{\varepsilon}) \]

By substituting (9) and (12) in (8), the total control force is calculated as below.

\[ u = -\frac{k_e}{\varepsilon} \left[ f(\dot{y}, y, t) + \lambda \dot{e} - \dot{y}_d \right] + -k_e \cdot \text{sat}(\frac{e}{\varepsilon}) \]

where \( \varepsilon \) is a positive constant which represents the boundary thickness, and sat(.) is defined as below.

\[ \text{sat}(\frac{e}{\varepsilon}) = \begin{cases} \frac{e}{\varepsilon} & \text{for } \frac{e}{\varepsilon} < 1 \\ \text{sgn}(\frac{e}{\varepsilon}) & \text{for } \frac{e}{\varepsilon} \geq 1 \end{cases} \]

4. SLIDING MODE FUZZY CONTROLLER DESIGN

The fuzzy logic controller (FLC) used in the method is proposed. The proposed FLC is based on Mamdani’s method. Below is the description of the fuzzy part of the sliding mode fuzzy controller.

**Fuzzification:** First, inputs and outputs of the fuzzy controller are determined. Second, an appropriate membership function (MF) should be selected.

**Fuzzy rule base:** Each rule is made of two parts. The first part is called antecedent which contains an inequality or inference which needs to be satisfied. The second part is called consequent which can conclude and is the output if the antecedent is satisfied. An example is shown below [25]:

**FR:** If \( A \), then \( B \)

where \( A \) is the antecedent and \( B \) is the consequent.

**Aggregation of the rule outputs:** The act of finding overall conclusion from consequents made by each rule is called aggregation. Max-Min aggregation method is used on this experiment which the calculation is described as bellow:

\[ \mu_F(x_k, y_k, u) = \max \left\{ \min_{r=1}^{r_{max}} \left[ \mu_R(x_k, y_k) \cdot \mu_F(x_k, y_k, u) \right] \right\} \]

where \( x_k, y_k \) are the input rules, \( r \) is the number of activated rules, and \( \mu_R(x_k, y_k) \) are fuzzy equivalents of the antecedent parts of the activated rule [ HYPERLINK 'Kov06' 26 ].

**Defuzzification:** The process which changes fuzzy output set to crisp output value is called defuzzification. Various types of defuzzification have been introduced. The center of area (COA) principle is used in this examination. In COA principle, crisp output is calculated as below:

\[ \mu_{FC}(x_k, y_k) = \frac{\sum_i \mu_i(x_k, y_k, u_i)}{\sum_i \mu_i(x_k, y_k, u_i)} \]

where \( \mu_i(x_k, y_k, u_i) \) is the membership function, \( \mu_{FC}(x_k, y_k) \) is the crisp output value, \( u_i \) is an element of the output fuzzy set [26].

The total control effort for the proposed model-free sliding mode fuzzy controller is:

\[ u_{tot} = u_{dis} + u_{fuzzy} \]

where \( u_{fuzzy} \) is the output of the fuzzy controller.

5. APPLICATION OF SLIDING MODE FUZZY CONTROLLER TO BUS SUSPENSION SYSTEM

The suspension control is widely used in the automotive industry. The main goal of designing a SMFC is to have quick response, no oscillation, and quick settling time. To prove the effectiveness of the proposed controller on the bus suspension systems, an experiment has been done in MATLAB/SIMULINK. Figure 2 illustrates SMFC implemented on the bus suspension system.

![Block diagram of the sliding mode fuzzy controller implemented on the bus suspension system](image-url)
The main role of the controller is to find an appropriate control law which would cause the bus suspension system to track reference trajectories. The controller parameters for the bus suspension system are defined as:

The sliding mode controller has error \( e \) as its input and \( u_{sl} \) as its output \( \lambda = 1.8897 \times 10^2, k_2 = 0.2589 \).

The fuzzy controller has one input \( S \) and one output \( u_{fuzzy} \). The input \( S \) is the output of the discontinuous part of the sliding mode controller. The membership functions of the fuzzy controller are illustrated in Fig 3.

![Figure 3: a) Membership functions of the input (S) of the fuzzy controller](image)

![Figure 4: b) Membership functions of the output (u_{fuzzy}) of the fuzzy controller](image)

In figure 3, the fuzzy subset of input and output linguistic variables are expressed as Negative Very Big (NVB), Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Big (PB), Positive Very Big (PVB).

The fuzzy rule base used here is composed of only 9 rules which are shown in table 2. The fuzzy rules have been decreased to reduce the calculation efforts. Two fuzzy rules in this controller are [25]:

\[ FR^1: \text{if } e \text{ is NB and } S \text{ is NB, then } U_f \text{ is NB} \]

\[ FR^2: \text{if } e \text{ is PB and } S \text{ is NB, then } U_f \text{ is ZE} \]

<table>
<thead>
<tr>
<th>Method</th>
<th>Settling time (sec)</th>
<th>Overshoot</th>
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<tbody>
<tr>
<td>Uncontrolled</td>
<td>46</td>
<td>0.92</td>
</tr>
<tr>
<td>PID</td>
<td>8</td>
<td>0.82</td>
</tr>
<tr>
<td>FLC</td>
<td>3</td>
<td>0.195</td>
</tr>
<tr>
<td>SMFC</td>
<td>1</td>
<td>0.04</td>
</tr>
</tbody>
</table>
From Table 3 and Figure 4, it is observed that SMFC has better settling time (1 sec) compared with PID controller (8 sec) and FLC (3 sec). It is also observed that SMFC has slightly better overshoot compared with PID controller, FLC, and uncontrolled model.

The obtained results from simulation prove that SMFC has the following advantages compared with other controllers:

- SMFC has much lower integral absolute error
- More vibration is absorbed by SMFC.
- As shown in Fig 5 and table 3, SMFC has lower vertical displacement overshoot
- SMFC has lower settling time

7. CONCLUSION
In this paper, a bus suspension system is modeled in MATLAB/SIMULINK. A sliding mode fuzzy controller (SMFC) has been designed for the suspension system. The proposed controller has diminished the effect of the chattering problem in the conventional SMC by using the boundary layer method. The model dependency of the SMFC was eliminated by removing the equivalent dynamic part of SMFC. The stability of the system was analyzed via SMC concept. The controller was designed and implemented on the suspension system to demonstrate the effectiveness of the controller. The results of SMFC were compared with the results of PID controller and Fuzzy logic controller. The results indicate that SMFC is an effective control method for bus suspension system.

REFERENCES